Strong heating of the soil : simulation of thermal convection-diffusion with phase-change in a saturated porous medium.*

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* ARPHYMAT Project: an interdisciplinary work in Mathematics, Physics, Archæology

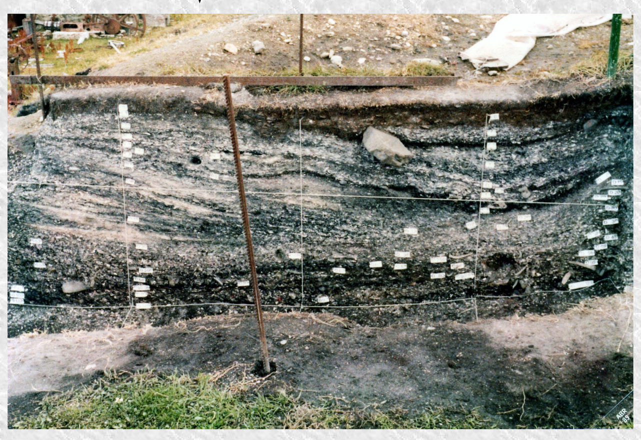
Context

Fires was used by prehistoric human groups for cooking food or potteries.

The study of these archæological hearths allows to understand the social behavior of these groups.

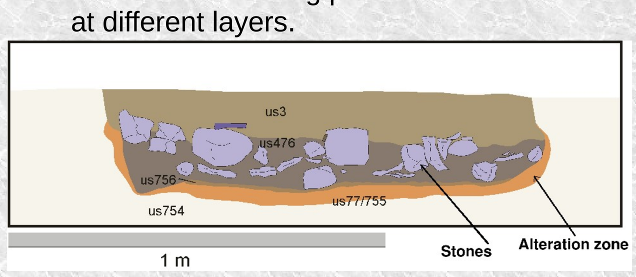
Cooking on hot stones. (today replication)

Fire and human societies:
– shape of the occupations ?
– mode of functioning ?
– utility ?
– minimum duration of burning ?



Alteration zones (at least 290 °C for limon-clay soils) reflects the temperature history during the burning.

Excavation showing prehistoric hearths at different layers.

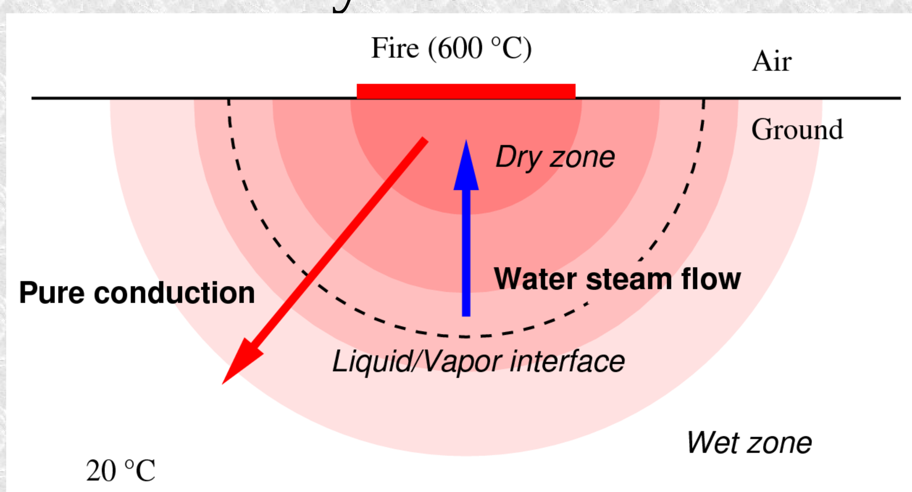


Aims:

➪ 3D-axisymmetric simulation of forced water evaporation in the soil.

➪ Retrieve information about the temperature history.

Physical model



water steam (low temperature in comparison to the fire) flows upward.

Assumptions:
- axisymmetric problem
- no gravity
- no capillary forces

Equations:
- energy conservation for the two components
- mass conservation for the water (either under liquid or vapor phase)
- Darcy law for the water steam in the soil
- ideal gas law (approx. near the evaporation pt.) for the behavior of the water steam

N.B.: A previous model (Ferreri, 1985) did not take into account the water steam flow.

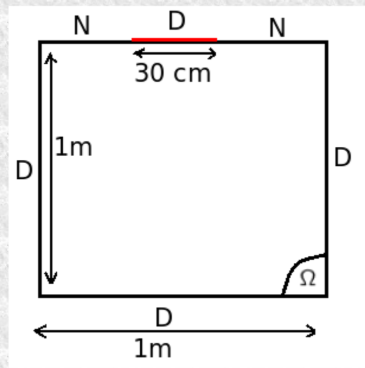
➪ Drawback of this model: only the heating stage can be simulated; for the cooling stage, non condensible gas (air) replaces water in the soil.

In the real situation, diffusion (downwards) and convection (upwards) are competitive:
➪ we can expect that taking into account the steam flow delay the heating process.

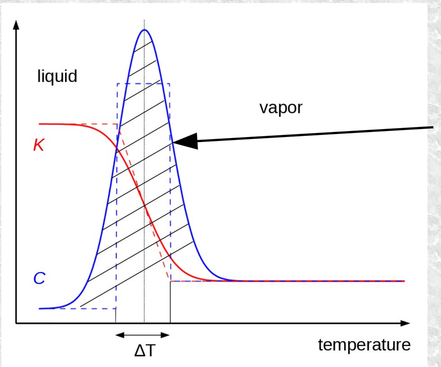
Question:

➪ Does coupling play an important role ?

Discretization and numerical methods



Computational domain Ω and boundary condition types *D* or *N*



Heat capacity *C* is modified to embed the latent heat.

Hatched area = latent heat *L*

C and *K* becomes continuous functions at the evaporation point.
(The same is true for viscosity and density.)

Apparent Heat Capacity method
(Bonacina *et al.*, 1973):
no front tracking

Use of a method of lines: only spatial discretization (**Finite Volume Method**) is made by hand.

$$\begin{cases} \dfrac{\partial T}{\partial t} = f(t,x,T,P) \\ \gamma\dfrac{\partial T}{\partial t} + \theta\dfrac{\partial P}{\partial t} = g(t,x,T,P) \end{cases}$$

Global system in terms of *T* (temperature) and *P* (pressure).

This PDE is algebraic because θ (related to the derivative of density wrt pressure) may vanish.

Automatic derivation of the Jacobian matrix via MAPLE.

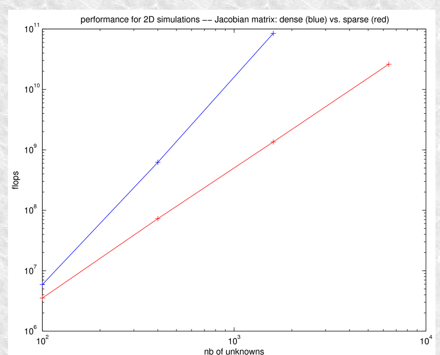
Use of an **automatic DAE solver** (variable step and order): DASSL (Petzold, 1990) of SLATEC

Towards performance and reliability

➪ Sparse Jacobian matrix (reduce size storage)

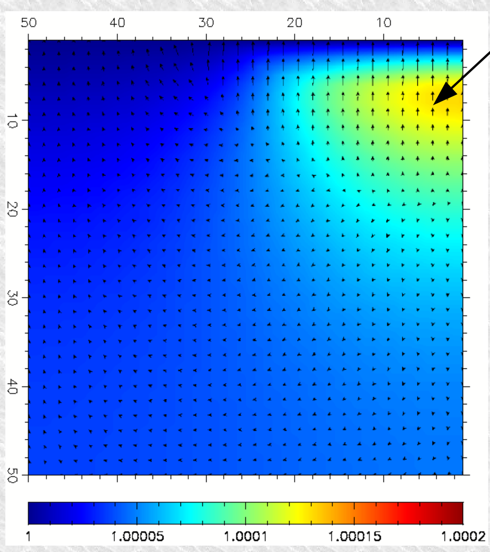
➪ Use of sparse solver (UMFPack) for the Newton iteration (drastic reduction of CPU time)

For a 5000 unknowns problem (50 x 50 grid, with two unknowns at each node):
- 6 hours of CPU time (dense Jacobian)
- about 10 minutes in the sparse case.



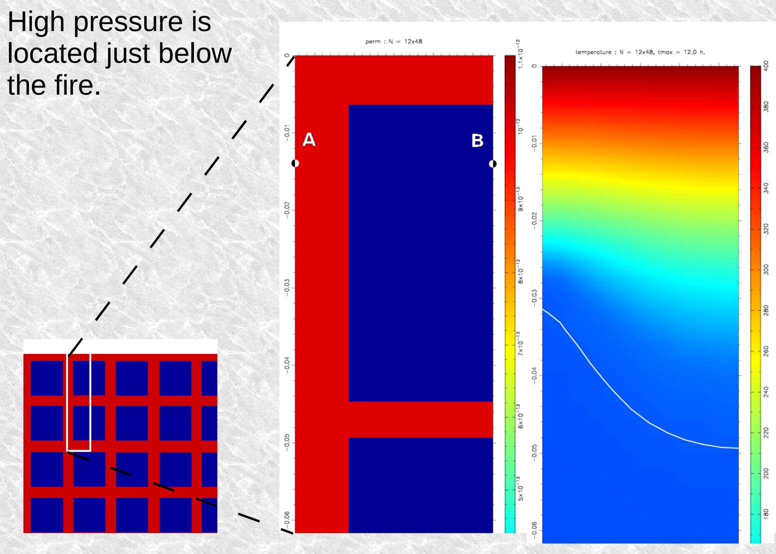
CPU time reduction when using a sparse Jacobian matrix.

Numerical results



High pressure is located just below the fire.

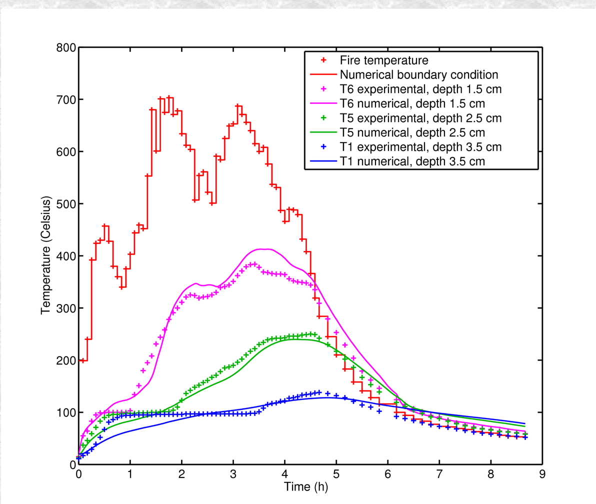
Homogeneous porous medium: Pressure and steam flow



Heterogeneous case: tiling square blocks Permeability and Temperature (in white: water/steam interface)

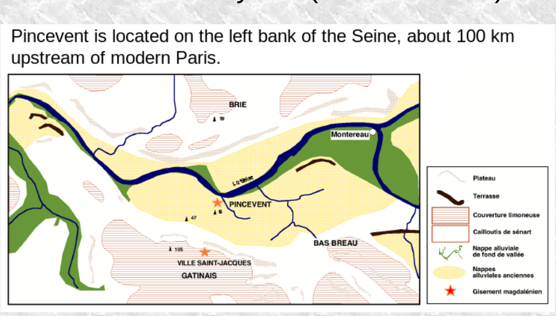
➪ A fully 3D numerical model is in progress, based on a non structured MHFE method.

Comparison with experiments



Temperature histories (simulation vs. experiment)
N.B.: the (quasi-systematic) presence of a plateau at 100°C is an open question.

Replication of an archæological fire on a limon-clay soil (Pincevent site).

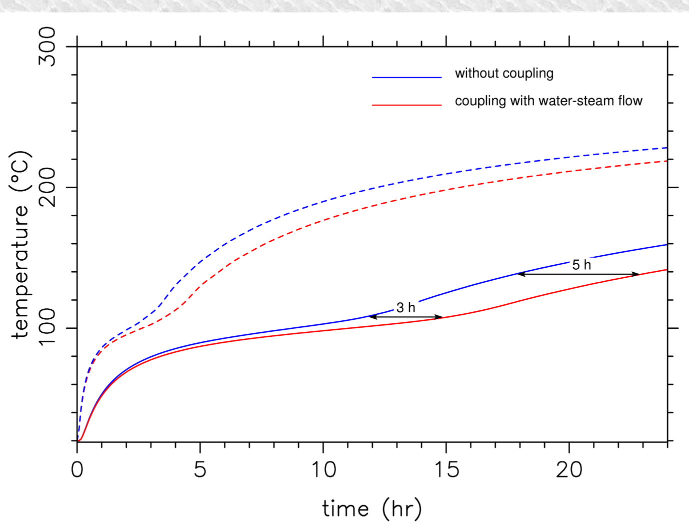


Pincevent is located on the left bank of the Seine, about 100 km upstream of modern Paris.

Conclusion:

The upward water steam flow plays an important role (figure on right):
It **can delay the heating of the soil** up to few hours, according to the depth of the considered point.

➪ It is crucial to take into account the convection part, when estimating the burning duration of prehistoric hearths.



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